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Comparison of FEA and analytical methods for determining stability of a RAP supported MSE wall

Emily C. Reed* **and Daniel R. VandenBerge**

Global stability is one of the failure modes that must be analysed for retaining walls. Limit equilibrium analysis of walls using slope stability software tends to result in a factor of safety that is either too high (circular surfaces) or too low (V-shaped non-circular surfaces). Finite element analysis (FEA) of walls provides a better solution but can be time-intensive and expensive. The primary aim of this project is to compare the results of FEA models with a simpler analytical bearing capacity method that uses Meyerhof's load inclination correction factors. In particular, cases were examined where Rammed Aggregate Pier reinforcing elements (RAPs) support a mechanically stabilised earth (MSE) retaining wall. For this project, several FEA models replicating these cases were created. Geometric parameters included the area ratio of RAP to matrix soil, or "replacement ratio", and the dimensions of the MSE wall. Each geometric configuration was then iterated over a range of undrained strength for the matrix soil, resulting in a different factor of safety for each model. A spreadsheet was also created containing the necessary calculations for the Meyerhof bearing capacity method. The factor of safety from the Meyerhof method was compared to the factor of safety computed for each corresponding FEA model. The results show an excellent relationship between the computed factors of safety for FEA models and the bearing capacity method, especially for factors of safety ranging from 1 to 1.5. At factors of safety above about 1.5, the critical failure mode becomes sliding rather than global stability, and the two methods diverge. The major implications of this research are that a complex FEA model can potentially be replaced by the simpler analytical Meyerhof bearing capacity method. Wall designers will benefit from a quick check on the global stability of a retaining wall without having to spend the time and money on more expensive FEA modelling.

Keywords: Rammed Aggregate Piers, Global Stability, FEA Modeling, Meyerhof

Introduction

Slope instability is a common engineering problem that may be solved with a variety of techniques. Slope stability is by definition exactly what one would expect: preventing soil slopes from moving away from their original designed placement. However, the failure mechanisms that can cause slopes to fail can be complex. This study focuses on the stability of mechanically stabilised earth (MSE) retaining walls which are one means of maintaining slope stability in locations where significant grade changes are required over short distances. The MSE walls examined in this study are supported by a foundation zone improved using Rammed Aggregate Piers (RAPs).

MSE walls must be designed to include an appropriate factor of safety against global instability. A key problem that arises when calculating the factor of safety involves deciding which method is the most accurate and applicable for the soil conditions. Some of the most accurate and versatile solutions come through the use of finite element analysis (FEA). However, in many cases, the resources are not available to perform an FEA study for every wall that is designed. This study, performed at the request of Geopier Foundation Company, seeks to find a relationship between FEA models and analytical

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methods to calculate factors of safety for global stability more efficiently.

Background

MSE walls

Mechanically Stabilized Earth (MSE) is a general term for reinforced soil placed in multiple layers (Federal Highway Administration 2009). Reinforced soil can be used for both reinforced slopes and retaining walls. A reinforced slope rises at an incline not exceeding 70 deg and requires two zones of reinforcement layers: a dense zone near the face of the slope and a well-spaced zone across the cross sectional width of the slope. Retaining walls with reinforced soil will rise at an angle exceeding 70 deg and tend to have only one zone of reinforcement (Federal Highway Administration 2009). These walls are usually backfilled with a coarse-grained soil. The geotechnical design at work in these walls is often overlooked by the public because the soil is hidden behind a "facing" usually made of decorative concrete panels or blocks. These walls are often chosen over a typical reinforced concrete retaining wall because they can be much more cost effective. Global stability is one important failure mode that must be considered for MSE walls. Global stability is a rotational type of failure mechanism that passes through the retained backfill, behind the reinforced zone, and through the foundation soil.

Rammed Aggregate piers

Rammed Aggregate Piers have been found to be an efficient means of improving global stability of MSE walls built over soft soils. The piers are installed by drilling a hole of desired length into the ground and then filling the hole with highly compacted aggregate. This aggregate is placed in lifts and each lift is hydraulically rammed by a bevelled tamper. The area replacement ratio is selected by designers based on the available capacity of the piers. The replacement ratio is equivalent to the ratio of the volume (or area in plan view) of the existing matrix soil that is replaced by the piers to the total volume of the reinforced soil. For example, a 22% replacement ratio means that 22% of the total volume of matrix soil will be replaced by piers.

Modelling of RAP is inherently a three dimensional problem because of the spacing of the piers. However, it is very common to convert these plans from a 3D plan view to a 2D elevation view; or plane strain condition, for modelling purposes. This simplification is typically justified because 3D modelling is much more time consuming and requires significantly more modelling expertise (Ariyarathne, Liyanapathirana, and Leo 2013). The 2D elevation view converts the piers into equivalent panels that represent the replacement ratio. This study will use the equivalent panel method.

Meyerhof bearing capacity

Global stability of retaining walls is closely related to bearing capacity (VandenBerge 2017). The analytical method used in this project to evaluate global wall stability was the Meyerhof bearing capacity method. The key difference in this bearing capacity method compared to others, such as those proposed by Terzaghi, is its capability to compute load inclination factors in an analytical manner. The Meyerhof method requires calculation of factors for bearing capacity, depth, and load inclination.

The bearing capacity factors related to the friction angle of the soil, ϕ', and were determined as follows (Meyerhof 1963):

$$N_c = (N_q - 1) * \cot \phi' \quad (1)$$

$$N_q = e^{\pi \tan\phi'} N_{\phi'} \quad (2)$$

$$N_\gamma = (N_q - 1) \tan(1.4\phi') \quad (3)$$

where,

$$N_{\phi'} = \tan^2 \phi' \left(\frac{1}{4}\pi + \frac{1}{2}\phi'\right) \quad (4)$$

The depth factors were calculated based on the bearing capacity factors above and the foundation geometry and were determined as follows (Meyerhof 1963):

$$d_c = 1 + 0.2\sqrt{N_{\phi'}} D/B \quad (5)$$

$$d_q = d_\gamma = 1, \ (\text{For } \phi' = 0) \quad (6)$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{N_{\phi'}} D/B, \ (\text{For } \phi' > 10^\circ). \quad (7)$$

Finally, the inclination factors were calculated based on the load inclination, α, and the friction angle, ϕ' and were computed as shown below (Meyerhof 1963):

$$i_c = i_q = \left(1 - \frac{\alpha^\circ}{90}\right)^2 \quad (8)$$

$$i_\gamma = \left(1 - \frac{\alpha}{\phi'}\right)^2. \quad (9)$$

After calculating the various factors required, they were incorporated into the general from of the Meyerhof ultimate bearing capacity equation given as (Meyerhof 1963):

$$q_{ult} = \frac{Q}{B} = d_c i_c c N_c + d_q i_q \gamma D N_q + \frac{1}{2} d_\gamma i_\gamma \gamma B N_\gamma. \quad (10)$$

Methodology

This section presents the two methods used to evaluate global stability for each geometric configuration of MSE wall and RAP that was studied. These methods include FEA modelling and application of the Meyerhof bearing capacity method to MSE wall stability.

FEA modeling approach

To study the relationship between FEA models and analytical methods, a series of FEA models were created in a programme called RS2, which is a part of the Rocscience software suite. The models were discretized using from 3200 to 5500 eight-noded quadrilaterals elements, depending on the geometric configuration. The boundary conditions for the model utilised both pins and rollers. The bottom of the model was assumed to be pinned across the entire length, and the outer edges of the model were assumed to be rollers. Figure 1*a* shows these boundary conditions in application. Figure 1*b* is a zoomed in view of the FEA model showing the mesh density used during discretization. The zones containing the MSE wall and the piers are more densely meshed because they are the point of interest within the model.

As Fig. 1 reveals, there are different zones indicating different materials used in each FEA model. The zones are as follows: matrix soil, MSE wall, engineered fill, and the four equally sized discrete panels used to represent the piers. The FEA models have geometric variations that include changing the ratio of RAP to matrix soil or "replacement ratio", and altering the dimensions of the MSE wall. The geometric variations are summarised in Table 1. The undrained strength, s_u, and modulus of elasticity, E, of the matrix soil are the other variables changed in the models. The range of values used are listed in Table 2. Every iteration, or "run" of the model maintained a conservative E/s_u ratio of 100 (e.g. Duncan and Mokwa 2001). All soils are considered isotropic and used the Mohr-Coulomb failure criteria. The remainder of the soil parameters used in the FEA models can be found in Table 2. The full height of the wall and backfill are reached in incremental stages to represent realistic staged construction techniques. Each stage is approximately 2 feet tall and each model has 10 stages. After the models were "built", strength reduction analysis was performed to determine the critical strength reduction factor (SRF) or factor of safety of the system.

Global stability of MSE walls was the primary focus of this study. When limit equilibrium slope stability methods are used for this purpose, a high strength zone is commonly used in lieu of explicitly modelling the reinforced fill. This approach effectively forces only global failures to occur. A similar approach is selected for this study. The MSE wall is treated as a uniform zone within the model with high shear strength, rather than modelling the reinforcement and its interaction with the reinforced soil explicitly. As a consequence of this approach, the strength of the MSE wall zone was not reduced during the FEA strength reduction analysis, which explains why its constitutive behaviour is indicated as "Elastic" only in Table 2. This approach assumes that appropriate checks on other failure mechanism are performed in separate analyses.

Meyerhof bearing capacity analytical approach

A spreadsheet was created containing the necessary calculations for the analytical Meyerhof bearing capacity method. The bearing capacity approach uses of a composite zone instead of the discrete zones used to represent piers in the FEA models. In the composite zone, the properties of the RAPs and the matrix soil are combined to calculate a single set of shear strength parameters and a single unit weight that is applied to the composite zone. The composite soil parameters are calculated as follows (Geopier Foundation Company 2016):

$$\phi'_{comp} = \tan^{-1}\left(R_a \tan\phi'_{pier} + (1 - R_a)\tan\phi'_{matrix}\right) \quad (11)$$

$$c'_{comp} = (1 - R_a)s_u \quad (12)$$

$$\gamma_{comp} = R_a\,\gamma_{pier} + (1 - R_a)\gamma_{matrix} \quad (13)$$

The composite zone is used only in the analytical calculations, which cannot account for discrete piers in a 3D array, and not in the FEA models.

Another difference in the analytical approach is that the factor of safety for bearing capacity is traditionally computed as a function of load:

$$F_{load} = \frac{\text{ultimate bearing capacity}}{\text{applied bearing pressure}} \quad (14)$$

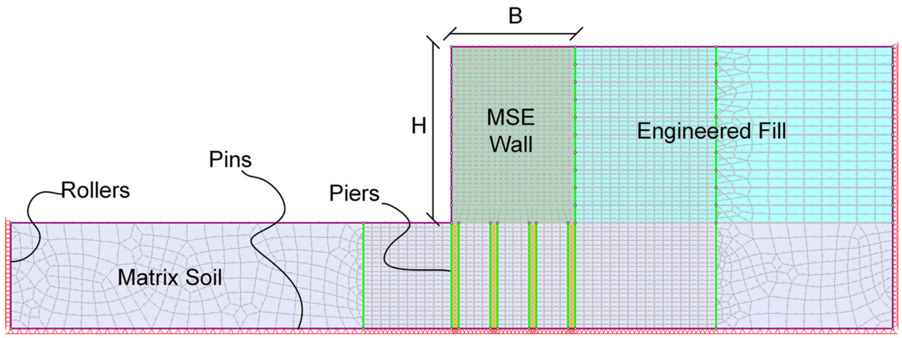
Whereas, global stability and the FEA strength reduction analysis calculate a factor of safety based on the strength of the soil:

$$F_{strength} = \frac{\text{shear strength of soil}}{\text{shear stress required for stability}} \quad (15)$$

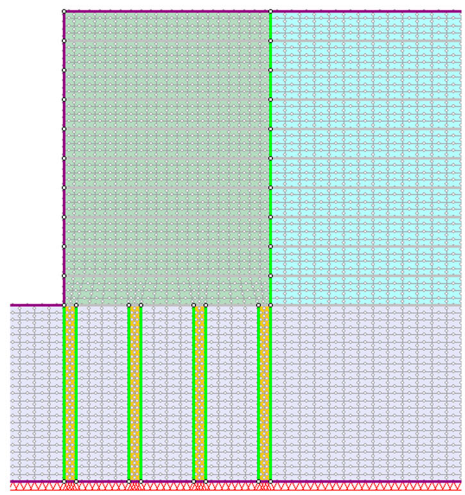
F_{load} is not equal to or proportional to $F_{strength}$ because ultimate bearing capacity is not linearly related to soil shear strength. In order to use the latter definition, the bearing capacity inputs were factored down to determine mobilised (or working) strength instead. This was done by iteratively dividing the shear strength parameters (c' and $\tan\phi'$ or s_u) by F until the applied bearing pressure equaled the mobilised bearing capacity based on the mobilised parameters, c'_{mob} and $\tan(\phi'_{mob})$ for the composite zone below the MSE wall and $s_{u,mob}$ for the matrix soil in front of the wall. This method results in a consistent definition of the factor of safety in terms of shear strength (Eqn. 15).

Similar to VandenBerge (2017), the base of the MSE wall was treated as the "foundation" for the bearing capacity analysis with the load applied at the base of the wall resulting from the weight of the MSE. In addition, the retained backfill applies forces to the wall zone. The mobilised vertical and horizontal forces on the back of the MSE wall are calculated using Coulomb earth pressure theory and the mobilised backfill friction angle. The applied bearing pressure in Eqn. 14 is calculated by dividing the vertical force at the base of the MSE wall (wall weight plus vertical backfill reaction) by the width of the wall, B. The applied bearing pressure is assumed to be uniform. The spreadsheet also allows for the application of a correction for the eccentricity caused by the backfill loading on the wall. The correction employs a simple reduced area approach that places the centroid of the base reaction at the middle of the reduced width of the wall, B*.

The potential failure surface will pass through both the matrix soil and the composite zone. The bearing capacities for the matrix soil and for the composite zone are calculated using Eqn. 1–10. The spreadsheet method also predicted the approximate failure surface geometry predicted by the Meyerhof approach (Reed 2018), assuming wedge-shaped active and passive zones connected by a radial Prandtl zone. The failure surface geometry was used to calculate the percentages of the failure surface in the composite zone and the matrix soil, respectively. A weighted-average bearing capacity was then calculated from these



(a) Boundary conditions and material zones



(b) Zoomed in to show mesh configuration

1 Boundary conditions and mesh configuration in a typical model

percentages. An example of the predicted failure mechanism is shown in Fig. 2.

Results

Previous research indicates that the theoretical failure surface drawn using the Meyerhof method corresponds well with the failure mechanism predicted by the FEA models. One example is provided in Fig. 2, where the theoretical failure surface is overlaid onto the FEA results. The contours in the figure represent maximum shear strain. The SRF for this particular model is 1.3 but maximum shear strain contours are shown for an SRF of 1.31. A slightly higher SRF aids in showing the failure surface location more clearly. It can be observed that the predicted failure surface shown with the dark line is fairly similar to the actual surface generated in the FEA model. It is evident that the failure mechanism is neither circular nor V-shaped failure surfaces as is commonly produced by limit equilibrium slope stability software for global stability calculations (VandenBerge 2017).

Table 1. Geometric variables

Geometry ID	Replacement Ratio, R_a (%)	Wall Height (ft)	Wall Width (ft)
3	22.9	20	14
4	30	20	14
5	15	20	14
6	5	20	14
7	20	20	14
8	5	50	35
9	15	50	35
10	20	50	35
11	22.9	50	35
12	30	50	35

*Geometry 1 and 2 are not shown and were used for preliminary purposes only.

Results from both the FEA models and Meyerhof calculations were compiled and analysed for comparison. The various geometric configurations studied can be reviewed in the previous section. All results have been synthesised into a single plot shown in Fig. 2. The analytical results include correction for eccentricity. This plot shows the factor of safety from the analytical methods versus the factor of safety from the FEA model for Geometries 3–12. Geometry 1 and 2 were used for preliminary purposes only.

The relationship between the analytical method and the FEA method can be observed in the plot in Fig. 3. For factors of safety between 1 and 1.5, the correlation between the two methods results in an error not greater than 5%. However, for factors of safety above 1.5 the data points start to

Table 2. Soil parameters

Soil	Unit weight (pcf)	Shear strength	Elastic modulus (ksf)	Poisson's ratio	Constitutive Behaviour
Matrix soil	120	s_u = 250–2000 psf	12.5–400	0.47	Elastic-Plastic
MSE	120	c' = 0, ϕ' = 45 deg	3000	0.47	Elastic
Eng. Fill	120	c' = 0, ϕ' = 30 deg	1000	0.4	Elastic-Plastic
RAP	140	c' = 0, ϕ' = 45 deg	3000	0.3	Elastic-Plastic

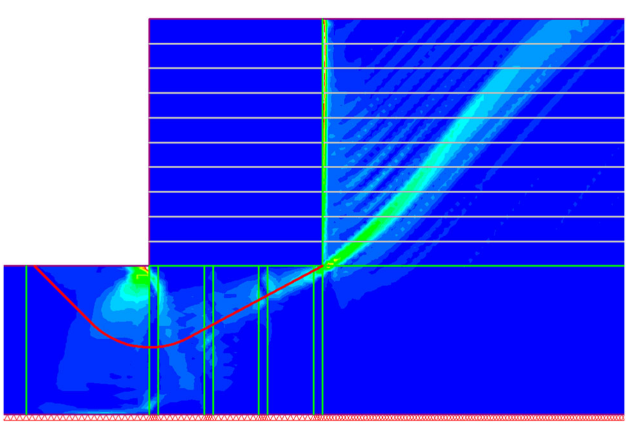
diverge from the 1:1 match line. A comparison of results with and without correction for eccentricity is given in Fig. 4. While the analytical method without the eccentricity correction more closely matches the FEA in most cases, the reason for favoring the use of the eccentricity correction is discussed in the next section.

Discussion of Geometry 5 Run 3

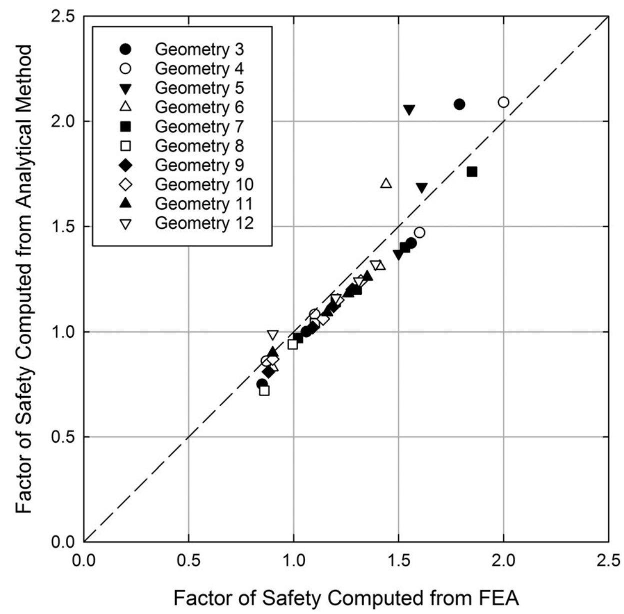
In order to better understand the outliers shown in Fig. 3, Geometry 5 was explored in more depth. Geometry 5 Run 3, with a s_u of 1425 psf, was of particular interest because it falls on the cusp of the transition from global stability to sliding as the controlling failure mechanism. This run also helps to illustrate the effects of eccentricity on the solution.

Without the eccentricity correction, the Meyerhof method resulted in a factor of safety of 1.92. This method was initially used for the entire project until was determined that eccentricity could be a potential reason for outlying data points. The only effect of the eccentricity correction on the analytical method is a change in the calculation of applied bearing pressure. For the method without eccentricity applied, the normal stress distribution along the bottom of the wall is assumed to be a constant stress acting across the entire width of the wall, B.

In comparison, the Meyerhof analytical method produces a factor of safety of 1.69 for this run when the reduced area correction for eccentricity is applied. The eccentricity correction distributes the vertical load only over the reduced wall width, B*. A visualisation of these two means of calculating the applied bearing pressure is given in Fig. 5.

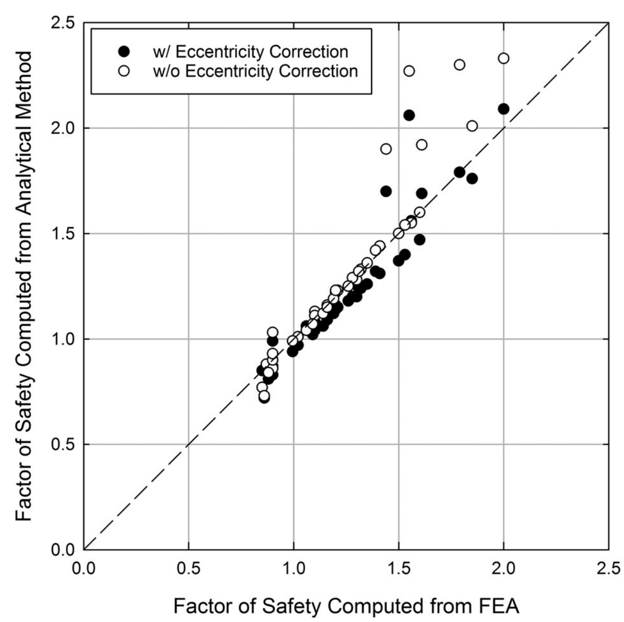


2 Theoretical failure surface comparison for Geometry 7 Run 3

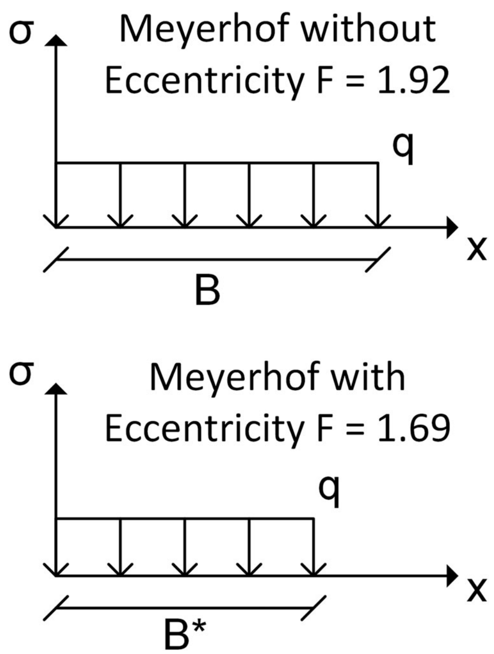


3 Results summary for all Geometries

The factor of safety resulting from the FEA model for Geometry 5 Run 3 is 1.61. This model used the same approach as all the other FEA models used in the project,



4 Comparison of results with and without eccentricity correction



5 Meyerhof analytical results for Geometry 5 Run 3 (B = 14 ft, B* = 11.76 ft)

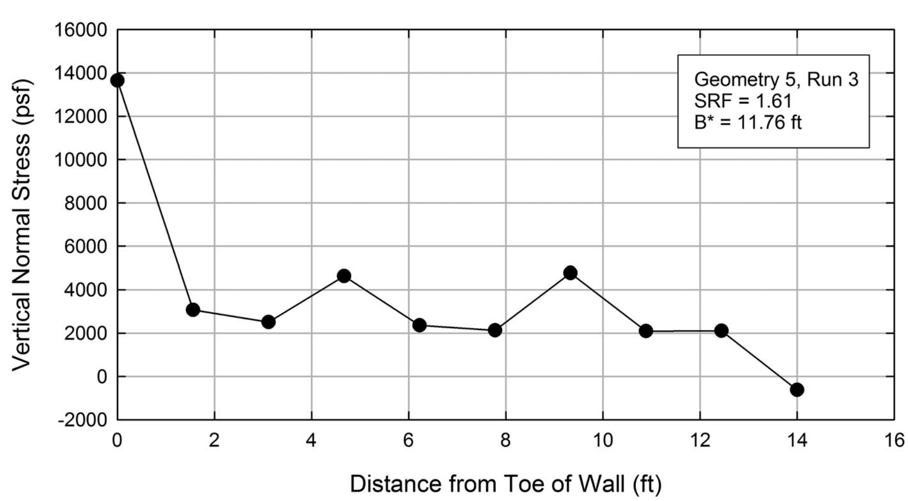
including staged construction and similar soil parameters. Because the FEA factor of safety had a poor comparison with the Meyerhof method (without the eccentricity correction), the normal stress distribution plot generated by the software was inspected and is shown in Fig. 6. This plot shows that the stress distribution is not consistent with the assumption of a constant stress acting at the bottom of the wall. Instead the distribution resembles a nearly triangular load with two spikes in the middle. These points of increased stress correspond to the location of the piers. Logically, these points of stress concentration make sense because stiffness attracts load and the piers are much stiffer than the surrounding matrix soil. It was hypothesised that the stress concentrations and the slight triangular shape of the load shown in Fig. 6 may be the cause of the difference the FEA and Meyerhof methods at higher calculated factors of safety.

In light of the stress distribution occuring in the Geometry 5 Run 3 FEA model, an analagous FEA model was created to further understand the model behaviour. For this additional FEA model, the wall and backfill were completely removed and replaced by equivalent distributed loads with magnitudes calculated from the thickness and unit weight of the wall and backfill. The vertical backfill load was applied as a constant distributed load across the width of the backfill section. This is depicted on the right side of Fig. 7. The horizontal component of the backfill load and the vertical wall load were combined into a single, inclined distributed load. The inclined load is shown on the left side of Fig. 7.

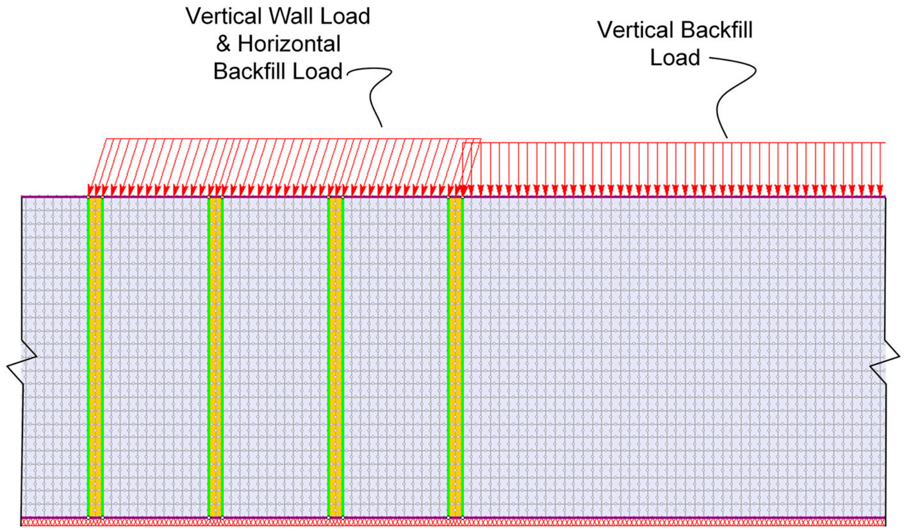
The resulting factor of safety from the FEA model with the modified loads shown in Fig. 7 is 1.73. This is a less conservative value than the one obtained from the initial FEA model for Geometry 5 Run 3. Table 3 shows a summary of the different analyses this configuration. When the vertical stress distribution in the FEA model more closely matches the constant vertical stress assumption of the Meyerhof method, the factors of safety are quite close, i.e. 1.69 vs. 1.73. When eccentricity is ignored, the analytical factor of safety becomes unconservative.

Fig. 8 plots the effective vertical normal stress just below the inclined load. The effective normal stress distribution in this figure is much different than the one generated by the full model (Fig. 6). The stresses closely resemble a uniform load and the stress concentrations on the piers are not as severe. The constrast in the vertical stress distributions between Fig. 6 and Fig. 8 strengthens the hypothesis that stress distribution at the base of the wall is playing a role in the outlying data points on the plot in Fig. 3.

Given all the information about the various methods used to analyze Geometry 5 Run 3, it is important to discern which methods are most helpful and which were used to better understand the model behaviour. Two approaches were discussed regarding the Meyerhof bearing capacity method. One method considered eccentricity, while the other neglected it. In practice, it is best to use



6 Stress distribution from FEA model



7 FEA inclined load modelling technique for Geometry 5 Run 3

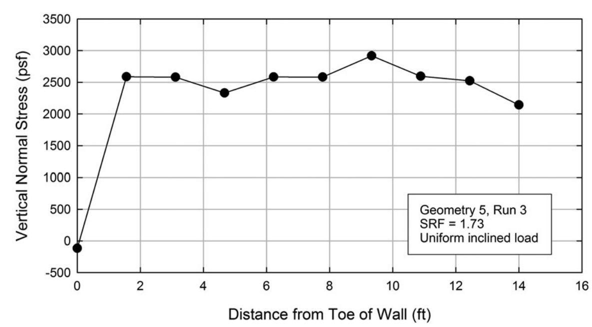
the approach that includes eccentricity because it leads to accurate or slightly conservative factors of safety compared to the FEA results.

As it happens, the eccentricities for this project had varying effects for the different models. At higher factors of safety, the eccentricity effects became much more pronounced and the two solutions begin to diverge. At factors of safety above about 1.5, it appears that interactions between the matrix soil, piers, and wall, including stress concentrations at the top of the piers, become too complex for the analytical bearing capacity method to accurately model. From a practical standpoint, this discrepancy was judged to be unimportant because the minimum required factor of safety (in terms of shear strength) for global stability is typically 1.5 or less.

As for the FEA models, two methods were explored as well. One method was the standard method used throughout the project and the other involved changing the method of load application to obtain a more uniform stress distribution. In order to get a realistic answer, the standard FEA modelling technique developed for this project is preferred. The FEA models with modified loading do not accurately represent the behaviour of the stress distributions under the wall. Additionally, the modified method gave a higher factor of safety which is less conservative and therefore more risky. The modified FEA model more closely matched the assumptions of the Meyerhof method and came very close to matching the factor of safety predicted by the Meyerhof approach. This indicates that the deviations between the two methods at higher factors of safety are caused by deviations from assumptions of the bearing capacity theory when the soil and RAP system becomes relatively quite strong.

Table 3. Geometry 5 Run 3 results summary

Analysis Method	F or Critical SRF
Standard FEA model of wall and RAP	1.61
Meyerhof BC with eccentricity	1.69
FEA model with inclined distributed load	1.73
Meyerhof BC without eccentricity correction	1.92



8 Stress distribution with inclined load

Summary and conclusions

Evaluation of global stability using the FEA method and the Meyerhof bearing capacity analytical solution have been discussed in detail. The FEA models were two-dimensional and used the programme Rocscience RS2. The results were computed using a strength reduction factor. The Meyerhof method utilised a spreadsheet to calculate the factor of safety. The factor of safety from the Meyerhof method was compared to the FEA SRF value.

The original hypothesis of this project was that a close relationship may exist between FEA and analytical methods for computing global stability of retaining walls, specifically MSE walls supported by RAP. For factors of safety within the range of 1–1.5, this hypothesis appears to be correct. There is indeed a very strong relationship between the two methods as shown in Fig. 2. The data points fall slightly below the 1:1 match line shown on the plot. This occurs because the eccentricity correction to the Meyerhof method is slightly conservative and brings the data points down. This phenomenon is not concerning because is it only slightly more conservative. The Meyerhof bearing capacity approach can be used to evaluate the global stability of MSE walls with RAP reinforced foundations for conditions in which the factor of safety falls between 1 and 1.5.

Unfortunately, the hypothesis starts to break down when factors of safety go above 1.5, which can be observed in the data. The deviations at higher factors of safety appear to be the result of increased complexity in the soil-RAP-wall interactions as the foundation becomes stronger and stiffer. While it would be desirable from a theoretical standpoint to understand these deviations more fully, it should not practically change wall design because the differences occur above the typical minimum required factor of safety.

The major implications of this research are that a complex finite element model can potentially be replaced by the simpler analytical Meyerhof bearing capacity method, if the factor of safety is in the acceptable range. Wall designers will benefit from a quick check on the global stability of a retaining wall without having to spend the time and money on more expensive FEA modelling. Both methods appear to be an improvement on global stability analysis of retaining walls using limit equilibrium slope stability software because more complex failure mechanisms can be captured by both.

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